# Everest Trek

**Building Math**

**Integrating Algebra and Engineering**

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# The Building Math Project

www.buildingmath.org

The United States is faced with the challenge of increasing the workforce in quantitative fields (e.g., engineering, science, technology, and math). Schools and teachers play a pivotal role in this challenge. Currently, many students (mostly from underrepresented groups) are not graduating from high school with the necessary math skills to continue studies in college in these quantitative fields. Industries and colleges such as Tufts University and General Electric realize that more active participation in the PreK–12 system is needed, and have put together this innovative program to help teachers increase math and engineering content in the middle-school curriculum.

Algebra and engineering are critical fields that are worth combining. Algebraic reasoning acts as a foundation for higher levels of math learning in secondary and tertiary education, and introducing students to engineering is a way to show students how math is used in a discipline of study and a career path.

Building Math is a three-year project funded through the GE Foundation. One goal of the project is to provide professional development for middle-school teachers in math and engineering, and to explore alternative teaching methods aimed at improving eighth grade students' achievement in algebra and technology. Another project goal is to develop standards-based activities that integrate algebra and engineering using a hands-on, problem-solving, and cooperative-learning approach.

The resulting design challenges were tested by teachers in ten Massachusetts schools that varied in type (public, charter, and independent); location (urban, suburban, rural); and student demographics. Each set of three activities form one unit where the activities are embedded into an engaging fictional situation to provide meaningful contexts for students to use the engineering design process and mathematical investigations to solve problems. There are three units, and each unit takes about three weeks of class time to implement. The instructional materials include reproducible student pages, teacher pages, a DVD of classroom videos for teacher professional development, a poster showing the engineering design process, and a Java applet used as a computer model in one of the activities.

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# Organization and Structure of Everest Trek

Throughout this reproducible book, you will find a teacher guide page on the left followed by a student activity page on the right. Additionally, each page is labeled as either a student page or a teacher page.

| Teacher guide page | Student activity page |
| --- | --- |

Answers to all activities and discussion questions are found in the answer key at the back of the book.

**TIPS**, **EXTENSIONS,** and **ASSESSMENT** opportunities are labeled in gray boxes.

**INTERESTING INFO** is provided in unshaded boxes. This provides additional related information and resources that you may want to share with students.

The following labels are used to indicate whether you will be addressing the whole class, teams, pairs, or individuals:

**CLASS**

**TEAMS**

**INDIVIDUALS**

**PAIRS**

**Tables and Graphs**
Tables and graphs are numbered according to their order of appearance in each design challenge. Those beginning with 1 correspond to Design Challenge 1. Those beginning with 2 correspond to Design Challenge 2. Those beginning with 3 correspond to Design Challenge 3.

## BUILDING MATH: PEDAGOGICAL APPROACH, GOALS, AND METHODS

The approach of the Building Math program is to engage students in active learning through hands-on, team-based engineering projects that make learning math meaningful to the students (Goldman & Knudson).

The goal of the Building Math units is to encourage the development of conceptual, critical, and creative-thinking processes, as well as social skills including cooperation, sharing, and negotiation by exhibiting four distinctive methods (Johnson, 1997):

1. **Contextual learning**—Each Building Math unit involves students in a story line based on real-life situations that pose fictional, but authentic, design challenges. Design contexts invite students to bring ideas, practices, and knowledge from their everyday lives to classroom work. Students apply math skills and knowledge in meaningful ways by using math analysis to connect inquiry-based investigations with creating design solutions.
2. **Peer-based learning**—Students work together throughout the design process. The key to peer-based learning is high amounts of productive, on-task verbalization. By verbalizing their thoughts, students listen to their own and others' thinking. This allows them to evaluate and modify one another's thinking and defend their own ideas. Verbalization also contributes to more precise thinking, especially when teachers use effective questioning techniques to ask students to explain and analyze their and others' reasoning.
3. **Activity-based practice**—Building Math uses design challenges (design and construction of a product or process that solves a problem) to focus peer-based learning. Students conduct experiments and systematic investigations; use measuring instruments; carefully observe results; gather, summarize, and display data; build physical models; and analyze costs and trade-offs (Richards, 2005).
4. **Reflective practice**—Building Math activities include questions, rubrics, and self-assessment checklists for students to document and reflect on their work throughout each design stage. Teams summarize and present their design solutions to the class, and receive and offer feedback on others' solutions.

### References

Goldman, S., & Knudsen, J. Learning sciences research and wide-spread school change: Issues from the field. Paper presented at the International Conference of the Learning Sciences (ICLS). http://mmap.wested.org/pathways/support/ICLS_Document_Pathways.pdf

Johnson, S. D. (1997). Learning technological concepts and developing intellectual skills. International Journal of Technology and Design Education, 7, 161–180.

Richards, L. G. (2005). Getting them early: Teaching engineering design in middle-schools. Paper presented at the National Collegiate Inventors & Innovators Alliance (NCIIA). http://www.nciia.org/conf05/cd/supplemental/richards1.pdf

## ENDURING UNDERSTANDINGS

In their book, *Understanding By Design,* Wiggins and McTighe advocate that curricula can be built by identifying enduring understandings. An enduring understanding is a big idea that resides at the heart of a discipline, has lasting value outside of the classroom, and requires uncovering of abstract or often misunderstood ideas. The list below details the enduring understandings addressed in Building Math. Teachers can consider these to be the ultimate learning goals for the Building Math units.

### Engineering and Technological Literacy

1. Technology consists of products, systems, and processes by which humans modify nature to solve problems and meet needs.
2. Design is a creative planning process that leads to other useful products and systems.
3. There is no perfect design.
4. Requirements for design are made up of criteria and constraints.
5. Design involves a set of steps, which can be performed in different sequences and repeated as needed.
6. Successful design solutions are often based on research, which may include systematic experimentation, a trial-and-error process, or transferring existing solutions done by others.
7. Prototypes are working models that can later be improved to become valuable products. Engineers build prototypes to experiment with different solutions for less cost and time than it would take to build full-scale products.
8. Trade-off is a decision process recognizing the need for careful compromises among competing factors.

### Math

1. Math plays a key role in creating technology solutions to meet needs.
2. Mathematical models can represent physical phenomena.
3. Patterns can be represented in different forms using tables, graphs, and symbols.
4. Graphs are useful to visually show the relationship between two variables.
5. Measurement data are approximated values due to tool imprecision and human error.
6. Repeated trials and averages can build one's confidence in measurement data.
7. Mathematical analysis can lead to conclusions to help one make design decisions to successfully meet criteria and constraints.
8. Analyzing data can reveal possible relationships between variables, and support predictions and conjectures.

### References

International Technology Education Assocation (ITEA)(2002). *Standards for Technological Literacy* (Second ed.). Virginia.

Wiggins, G., & McTighe, J. (2005). *Understanding by Design* (2nd ed.): Prentice Hall.

## EVEREST TREK OVERVIEW: STORY LINE AND LEARNING OBJECTIVES

| | Design Challenge Overview | Students will: |
|---|---|---|
| **Design Challenge 1: Gearing Up!** | Students imagine that they are preparing to climb Mount Everest. They learn about the climate conditions and need to design a coat to protect them from the cold and wind. Students research the insulation performance of different materials to help determine which materials to choose for their design. The coat must keep the temperature above 65˚F for 30 seconds, not exceed a thickness of 2 cm, and be as low in cost as possible. | • Interpret a line graph.<br>• Locate and represent the range of acceptable values on a graph to meet a design criteria.<br>• Extrapolate data based on trends.<br>• Conduct two controlled experiments.<br>• Collect experimental data in a table.<br>• Produce and analyze graphs that relate two variables.<br>• Determine when it's appropriate to use a line graph or a scatter plot to represent data.<br>• Distinguish between independent and dependent variables.<br>• Apply the engineering design process to solve a problem. |
| **Design Challenge 2: Crevasse Crisis!** | As students are "climbing" Mount Everest, they come to a large crevasse. They study the sagging effect of loading weight onto bridges of different sizes and shapes to design a bridge that will enable them to safely cross the crevasse. The bridge must meet these criteria and constraints: It should support a minimum amount of weight without sagging more than a specific amount, use as few ladders as possible for construction, and be a minimum width to allow for safe crossing. | • Use proportional reasoning to determine dimensions for a scale model.<br>• Use physical and math models.<br>• Conduct two controlled experiments.<br>• Collect experimental data in a table.<br>• Produce and analyze graphs that relate two variables.<br>• Compare rates of change (linear versus non linear relationships).<br>• Distinguish between independent and dependent variables.<br>• Apply the engineering design process to solve a problem. |
| **Design Challenge 3: Sliding Down!** | Students have reached the top of Mount Everest! But now they face the problem of altitude sickness and a need to quickly transport their sick classmates down the mountain. Students experiment to find a relationship between the angle of a zip-line and the speed of moving along the zip-line down the mountain. Students use the results of their research to design a zip-line transportation device that meets these criteria and constraints: It must move within a range of acceptable speeds, be stable and secure, and include a return mechanism. | • Conduct a controlled experiment.<br>• Measure angles using a protractor.<br>• Compare and discuss appropriate measures of central tendency (mean, median, mode).<br>• Apply the distance-time-speed formula.<br>• Produce and analyze a graph that relates two variables.<br>• Locate and represent the range of acceptable values on a graph to meet a design criteria.<br>• Distinguish between independent and dependent variables.<br>• Apply the engineering design process to solve a problem. |

## ASSESSMENT OPPORTUNITIES AND MATERIALS LISTS

The tables on the next two pages list the opportunities for formative assessment by using rubrics, probing students' thinking during class time, and reviewing student responses to certain questions. The tables also show the materials needed for each design challenge. The numbers in the assessment column refer to the steps of the engineering design process.

| | Assessment | Materials |
|---|---|---|
| **Design Challenge 1: Gearing Up!** | • **2. Research:** Assess whether students can make a complete graph and correctly represent the experimental data. Use the rubric on page 148.<br>• **2. Research:** Assess whether students can describe the relationship represented in the graph by the variables in the *x*- and *y*-axes.<br>• **2. Research:** Assess whether students can extrapolate from the trend of the data.<br>• **2. Research:** Assess whether students can apply the findings of their research to create a rule of thumb to help meet design criteria.<br>• **4. Choose:** Assess engineering drawing based on quality and communication. Use the rubric on page 149.<br>• **6–8. Test, Communicate, Redesign:** Assess written responses and student observations during test, communicate, and redesign steps based on model performance, completeness, and quality of reflection. Use the rubric on page 152.<br>• **Individual Self-Assessment Rubric:** Students can use the checklist on page 77 to determine how well they met behavior and work expectations.<br>• **Team Evaluation:** Students can complete the questions on page 79 to reflect on how well they worked in teams and celebrate successes, as well as make plans to improve teamwork.<br>• **Student Participation Rubric:** Make copies of the rubric on page 153 to score each student's participation in the design challenge. | For each team:<br>• about 5 pieces (15 cm × 15 cm each) of each of the four fabric materials (denim, fleece, nylon, wool)<br>• digital thermometer<br>• 2 frozen ice packs<br>• stopwatch<br>• calculator<br>• 4 different-colored pencils or thin markers<br>• ruler<br>• chart paper<br>• pack of broad markers (different colors) |
| **Design Challenge 2: Crevasse Crisis!** | • **1. Define:** Assess how students use the scale to find the model measurements given the actual measurements.<br>• **2. Research:** Assess whether students can interpret the graph and how it relates the two variables represented by the *x*- and *y*-axes.<br>• **2. Research:** Assess whether students can interpolate data using the graph.<br>• **2. Research:** Assess whether students can apply the findings of their research to create a rule of thumb to help meet design criteria.<br>• **2. Research:** Assess whether students can make a complete graph and correctly represent the experimental data. Use the rubric on page 148.<br>• **4. Choose:** Assess engineering drawing based on quality and communication. Use the rubric on page 150.<br>• **5. Build:** Assess model/prototype artifact based on craftsmanship and completeness. Use the rubric on page 151. | For each team:<br>• 2 identical textbooks, packs of copy paper, or similar equally sized sturdy rectangular prisms<br>• a flat table to work on<br>• 10 pennies<br>• 2 pieces of foam that are each at least 30 cm × 17.5 cm<br>• 1 piece of foam that is at least 30 cm × 12 cm |

## ASSESSMENT OPPORTUNITIES AND MATERIALS LISTS *(CONTINUED)*

| | Assessment | Materials |
|---|---|---|
| **DC 2: Crevasse Crisis! *(continued)*** | • **6–8. Test, Communicate, Redesign:** Assess written responses and student observations during test, communicate, and redesign steps based on model performance, completeness, and quality of reflection. Use the rubric on page 152.<br>• **Individual Self-Assessment Rubric:** Students can use the checklist on page 113 to determine how well they met behavior and work expectations.<br>• **Team Evaluation:** Students can complete the questions on page 115 to reflect on how well they worked in teams and celebrate successes, as well as make plans to improve teamwork.<br>• **Student Participation Rubric:** Make copies of the rubric on page 153 to score each student's participation in the design challenge. | For each team:<br>• graph or white paper<br>• ruler<br>• scissors<br>• tape<br><br>For the class:<br>• 1 large cup with string attached<br>• 500 pennies |
| **Design Challenge 3: Sliding Down!** | • **2. Research:** Assess whether students can make a graph that contains all the parts and correctly represents the data using the rubric on page 148.<br>• **2. Research:** Assess whether students can interpret the graph and how it relates the two variables represented by the *x*- and *y*-axes.<br>• **2. Research:** Assess whether students can extrapolate data using the graph.<br>• **2. Research:** Assess whether students can apply the findings of their research to create a rule of thumb to help meet design criteria.<br>• **4. Choose:** Assess engineering drawing based on quality and communication using the rubric on page 150.<br>• **5. Build:** Assess model/prototype artifact based on craftsmanship and completeness using the rubric on page 151.<br>• **6-8. Test, Communicate, Redesign:** Assess written responses and student observations during test, communicate, and redesign steps based on model performance, completeness, and quality of reflection using the rubric on page 152.<br>• **Individual Self-Assessment Rubric:** Students can use the checklist on page 141 to determine how well they met behavior and work expectations.<br>• **Team Evaluation:** Students can complete the questions on page 143 to reflect on how well they worked in teams and celebrate successes, as well as make plans to improve teamwork.<br>• **Student Participation Rubric:** Make copies of the rubric on page 153 to score each student's participation in the design challenge. | For each team:<br>• 2 metersticks<br>• piece of fishing line 5 meters long<br>• piece of fishing line 2.05 meters long<br>• piece of straw 5 cm long<br>• protractor<br>• ruler<br>• stopwatch (measures at least to nearest tenth of a second)<br>• chart paper<br>• markers<br>• card stock (8.5 in × 11 in)<br>• scissors<br>• tape<br>• calculator<br>• small toy figures |

# Everest Trek Master Materials List

| Qty | Item | C* or R* | 1. Gearing Up! | 2. Crevasse Crisis! | 3. Sliding Down! |
|---|---|---|---|---|---|
| **PER GROUP** | | | | | |
| 1 | calculator | R | ✓ | | ✓ |
| 1 | piece of card stock (8.5” × 11”) | C | | | ✓ |
| 2 | sheets of chart paper | C | ✓ | | ✓ |
| 1 | pack of colored pencils or thin markers (min. 4 colors) | R | ✓ | | |
| 1 | piece of craft foam (30 cm × 12 cm) | C | | ✓ | |
| 2 | piece of craft foam (30 cm × 17.5 cm) | C | | ✓ | |
| 8 | craft sticks | C | | ✓ | |
| 5 | pieces of denim (15 cm × 15 cm) | R | ✓ | | |
| 1 | digital thermometer | R | ✓ | | |
| 1 | drinking straw (5 cm) | C | | | ✓ |
| 1 | fishing line (about 9 m) | C | | | ✓ |
| 5 | fleece (15 cm × 15 cm) | R | ✓ | | |
| 3 | graph or white paper | C | | ✓ | |
| 2 | ice packs | R | ✓ | | |
| 1 | pack of markers (broad, different colors) | R | ✓ | | ✓ |
| 2 | metersticks | R | | | ✓ |
| 5 | pieces of nylon (15 cm × 15 cm) | R | ✓ | | |
| 10 | pennies | R | | ✓ | |
| 1 | protractor | R | | | ✓ |
| 1 | ruler | R | ✓ | ✓ | ✓ |
| 1 | scissors | R | | ✓ | ✓ |
| 2 | small toy figures | R | | | ✓ |
| 1 | stopwatch | R | ✓ | | ✓ |
| 1 | tape (invisible) | C | | ✓ | ✓ |
| 5 | wool fabric (15 cm × 15 cm) | R | ✓ | | |
| **PER TEACHER** | | | | | |
| 1 | cup (large) with string attached | | | ✓ | |
| 500 | pennies | | | ✓ | |

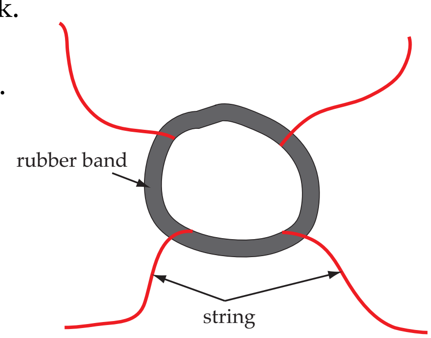
*C = Consumable
*R = Reusable

## CUP-STACK TEAM-BUILDING ACTIVITY

Students work and communicate in teams during most of each design challenge in *Everest Trek*. Some pilot teachers found it useful to do some team-building activities prior to the start of the unit. There is a different team-building activity in each of the Building Math units.

**Objectives**

- Students work together in teams to accomplish a timed task.
- Students practice communication skills.
- Students reflect on one's participation in a teamwork setting.




**Group Size:** 3 to 4 students

**Materials:** You will need a stopwatch or a clock/watch to time 1 minute. Each team needs 15 foam or plastic cups and a rubber band with strings attached (see setup instructions).

**Setup**

- Cut string into 2-foot lengths. Tie four strings to the rubber band evenly spaced around the circle. It should look like a sun with four rays coming out.
- Divide the cups into stacks of 15.

**Procedures**

1. Explain to the class that they will participate in a team-building activity that focuses on accomplishing a task and communication.
2. Distribute a set of materials to each team. Explain that the task is to build a pyramid using the cups within a 1-minute time limit. The pyramid will begin with 4 cups in a row at the base, 3 cups on the next row, then 2 and finally 1 cup at the top. Group members may not touch the cups with their hands or any part of their body, even if the cups fall. Each person may only hold the end of one string attached to the rubber band. Group members must work together to stretch and relax the rubber band to grab each cup and place the cup in the right place.
3. When groups are ready, start timing 1 minute. When 1 minute is up, stop the activity and check each team's progress.
4. Debrief the cup-stack activity with these questions:
   - Was anyone frustrated at all during the activity? If so, how was it handled?
   - Why is teamwork so important for this activity?
   - Did any team come up with a strategy for working together as a team? If so, what was the strategy?
   - Are you ever in a situation where you must use teamwork? Is it always easy for you? Why or why not?
   - What are some skills needed to be good at teamwork?
   - What is so hard about teamwork?
   - How did you contribute to your team? Did you give suggestions? Lead or follow? Encourage or cheer?
   - How would you do the activity differently if you were asked to do it again?
5. Reset and repeat the activity. Give teams a minute to strategize before starting the time. After the task, debrief with these questions:
   - Did your teamwork improve this time? How and why did it improve?
   - Why is good communication important to accomplishing this task?

## EVEREST TREK PRE-REQUISITE MATH SKILLS

Students will be using the math skills listed below while doing the Everest Trek design challenges. You may want to review these skills as a short warm-up exercise at the start of the class or as a homework assignment on the night before the challenge when the skill will be used. The skills in bold are reviewed in the activities on pages 19–30.

| STUDENTS SHOULD BE ABLE TO: | | | |
|---|---|---|---|
| MATH SKILL | 1. GEARING UP! | 2. CREVASSE CRISIS! | 3. SLIDING DOWN! |
| Interpret a line graph. | ✓ | ✓ | ✓ |
| **Make a line graph.** | ✓ | ✓ | ✓ |
| Use a ruler to measure length in centimeters (to the nearest millimeter). | ✓ | ✓ | ✓ |
| Round numbers to a given place value (including decimals and powers of 10). | ✓ | ✓ | ✓ |
| Multiply and add decimals (money) and whole numbers using a calculator or paper/pencil. | ✓ | | |
| **Use a given scale to determine scale model dimensions given actual dimensions.** | | ✓ | |
| Multiply and divide decimals by powers of 10. | | ✓ | |
| Measure angles with a protractor. | | | ✓ |
| **Use the speed = distance/time formula (given distance and time, find speed using a calculator).** | | | ✓ |

## EVEREST TREK PREREQUISITE MATH SKILLS

### How to Use the Activities

1. Line Graph Activity (pages 20–24)
    - The goal of this activity is for students to:
    - Make a line graph.
        - identify independent and dependent variables.
        - use convention to put the independent variable on the *x*-axis and the dependent variable on the *y*-axis.
        - use range of data to set up scales on axes so that the data is well spread out.
        - use equal intervals when setting up scales on axes.
        - label the axes with data type and unit.
        - label the graph with an appropriate title.
    - Use Exercise 1 to guide students through the steps of constructing a line graph—particularly steps 2 and 3 (scaling the axes).
    - If students need additional guidance in scaling the axes, use the Line Graph Scaling Examples on pages 25–28.
    - Assign students to do Exercise 2 on their own or with a partner.

2. Using a Scale Activity (pages 29–30)
    - The goal of this activity is for students to:
        - use a scale to determine scale model dimensions given actual dimensions.
        - use intuitive proportional reasoning (using drawings, multiplying or dividing by same number to maintain equal ratios).
        - solve proportions as two equal ratios using an equation.
    - Go over examples 1 and 2 the students.
    - Assign students to work through the exercises on their own or with a partner.