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The Fulton County Middle School Mathematics *Academic Support Program (ASP)* for summer school is a complete, turnkey solution for improving educational performance. Topics are built around accessible core curriculum, ensuring that the *ASP* is useful for striving students and diverse classrooms.

This program recognizes that many struggling students aren't reached by traditional "skill-and-drill," or strict test-prep approaches.

The *ASP* includes components that review, instruct as needed, provide practice, and assess students' skills. Instructional tools and strategies are embedded throughout. The scope and sequence addresses the needs of students who require additional support in topics included in the Georgia Performance Standards and the National Council of Teachers of Mathematics Standards.

This 3-ring binder includes the following:

- Over 20 days' worth of lessons with reproducible activity sheets keyed to standards
- A section on test-taking strategies and a collection of practice items
- A collection of station-based activities for small group work
- A problem-based mathematics teacher's guide that:
 - describes the purpose of the materials and options for using the program
 - provides pacing chart options
 - recommends an assortment of graphic organizers for instructional strategies
 - includes an assortment of openers—to begin a class or to make a transition
 - references relevant state standards

Purpose of Materials

The *Academic Support Program* for Middle School Mathematics is a flexible program that has been organized to fit your students' needs in summer school.

Each day's schedule begins with direct instruction and guided practice and moves on to opportunities for developing and applying new skills and concepts in problem-solving situations.

The twenty-day program includes all of the major content strands in the Fulton County Middle School Mathematics Curriculum and the Georgia Performance Standards. These include the following:

- Number and Operations
- Geometry and Measurement
- Algebra
- Data Analysis and Probability

Problem solving, reasoning and proof, communication, connections, and representation are infused throughout.

Structure of the Binder

The *ASP* is provided, for your convenience, in a binder format. The materials are completely reproducible, allowing you to make as few or as many copies as you need. If students lose an activity sheet, it's easy to make a new one. Tabs allow you to access the different sections of the binder quickly and easily.

The Teacher's Guide is the first section. It is organized to help you navigate the materials with the pacing guides. The Teacher's Guide offers fifteen graphic organizers and suggested strategies for their use, includes a collection of "openers" to start class, and shows how the lessons correlate to the Georgia Performance Standards and Grade Level Expectations.

The next four sections focus on content and knowledge of Number and Operations; Geometry and Measurement; Algebra; and Data Analysis and Probability. The units in the *ASP* can be implemented as prescribed in the pacing guides, yet the design is flexible so that you can mix and match sections and units as the needs of your students and your instructional style dictate.

Part 6 contains a collection of hands-on activities designed for cooperative learning at a series of classroom stations. These correspond to the four content strands, and give students the opportunity to apply concepts and skills while you have a chance to circulate, observe, speak to individuals and small groups, and informally assess and plan. You might choose to do a set of station activities each day, every other day, as a follow-up to direct instruction, or as a replacement for it.

The final section focuses on test taking. This section allows you to demystify tests, such as the CRCT, for your students. There are five practice assessments for each section of instruction. You may choose to administer some of the assessments as you move through the instructional sequence, or utilize them only at the end. Used on their own or in combination with the content-area units, students will gain confidence in their abilities through familiarity of question types within the context of the content they have reviewed.

Structure of Units

Nearly all the instructional units in Parts 2, 3, 4, 5, and 6 have some common features.

In each class session, you will present a topic. Some topics may be a review for students; other topics may be completely new to them. After some instruction, you will provide students with practice activities to try. Students will have a chance to talk about how they completed their work. There are additional materials to use if you are confident that students are ready to extend their learning. If students need more practice or further explanation, you can provide them with that, too!

1. The Goal Statement

Each unit begins with a brief objective of what students should know, understand, or be able to do at the end of the unit.

2. Words to Know

Vocabulary terms are provided as background information for instruction, or to review key concepts that are addressed in the unit.

3. Background

An explanation of the mathematics and/or instructional model is included for your information. This may include details about the history of math or about applying math skills in the real world.

4. Direct Instruction

This section is a guide for a teacher-led activity to review and/or instruct students on a specific skill or topic (activities are 15 to 45 minutes in length). Instructional strategies include lecture, modeling, discussion, group facilitation, and more. The activities often include the use of one or more of the graphic organizers found in the Teacher's Guide. This section frequently refers to presenting information on the board or via an overhead projector, and includes diagrams and sample problems to be presented to the class in this manner.

5. Student Activity Pages

Each unit includes three or more lesson tasks and activities to support students' achievement of learning objectives. These sheets are written for the student. They can be used in any combination of teacher-led instruction, cooperative learning, or independent application of knowledge.

Number Theory

Instruction

Goal: To understand and develop strategies for identifying factors and multiples

WORDS TO KNOW

composite number	a natural number that is not prime
factor	one of two or more expressions that are multiplied together to get a product
multiple	a multiple of a number is the product of that number and any other whole number; zero is a multiple of every number
prime number	a number whose only factors are itself and 1
prime factors	the prime numbers that divide into an integer exactly, without leaving a remainder
greatest common factor	the largest number that divides two or more numbers evenly
least common multiple	the smallest nonzero number that is a multiple of two or more numbers

Direct Instruction

Explain to students that concepts of factoring, finding multiples, and knowing whether numbers are prime or composite, are all tools that they'll need for working with fractions, doing algebra, and solving other kinds of problems.

Using tiles or a grid template on an overhead projector, show students the idea of prime and composite numbers and factors and multiples by demonstrating the ways that numbers can be displayed in graphic form.

For example, three tiles or squares have to be displayed in a single row while four tiles or squares might be in one row of four or two rows of two.

Class Activity

Materials: tiles or grid paper for students

1. Take six tiles and ask students to arrange them into possible areas. There are only two possible configurations. Both cases show the area is four square units—six tiles arranged in a row (6×1 rectangle) and three tiles arranged in two rows (3×2 rectangle). In this case, we could say numbers 1, 2, 3, and 6 are factors of 6 (6 is divisible by 1, 2, 3, and 6).

2. Explain to students that when a number such as 4 and 6 has *more* than two factors, it is called a **composite number**.
3. Now try five tiles. Students will place (or color in) five tiles in one row (5×1 rectangle). Point out that these numbers can only be divided by 1 and the number itself.
4. Explain to students when a number such as 3 and 5 has *only* two factors, it is called a **prime number**.

Class Activity: Diagramming Divisibility

Math topics

divisibility rules, multiples, factors, Venn diagrams

Prior knowledge needed

- divisibility rules for 2, 3, and 5
- multiples
- factors
- understanding of a three-set Venn diagram

Materials needed

- an activity sheet for each student
- one blank three-set Venn diagram for each pair of students

Engaging the students

Use a three-set Venn model and label one circle “students wearing jeans,” the second circle, “students wearing red shirts,” and the third circle, “students with brown hair.” Ask students to help you place each student in the correct area of the Venn diagram. Discuss whether each of the placements is correct and why. Explain to students that they are going to be using numbers to complete a three-set Venn diagram similar to the one you just did.

The exploration

Review the divisibility rules on page 4 before students begin the activity. When they have completed the activity, collect these sheets and assign partners to design an original three-set Venn diagram problem. Students should be permitted to choose from a variety of subjects including plant species, animal species, and so forth.

Debriefing

Allow time for each pair of students to share their original Venn diagram with another group. Time should be allotted for students to discuss their Venn puzzles and any changes that they would like to make in their choices.

Assessment

1. Student products: Assess student activity sheets and each group's original three-set Venn diagram problem.
2. Optional prompts for reflective writing or class discussion
 - a. Level 1 question: You are examining a Venn diagram that shows the multiples of 3 in one circle and the multiples of 7 in the other. There is no number in the middle region (where the two circles intersect). Give an example of a number that might be placed correctly in this region. Explain how you got your answer.
 - b. Level 2 question: You are examining a Venn diagram that depicts the multiples of 2, 5, and 7. There is no number in the center region (where all three circles intersect). Give an example of a number that might be correctly placed in this region. Explain your answer.

NAME:

PART 2 • NUMBER AND OPERATIONS
Number Theory

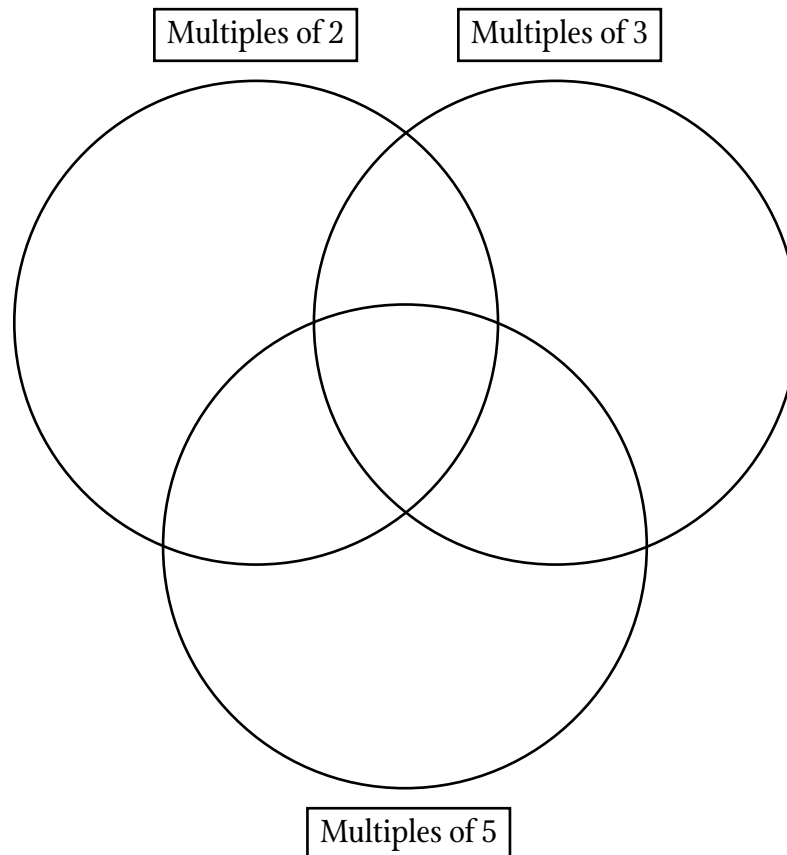
Divisibility Rules

Divisibility by:

2	If the last digit is even, the number is divisible by 2.
3	If the sum of the digits is divisible by 3, the number is also.
4	If the last two digits form a number divisible by 4, the number is also.
5	If the last digit is a 5 or a 0, the number is divisible by 5.
6	If the number is divisible by both 3 and 2, it is also divisible by 6.
7	Take the last digit, double it, and subtract it from the rest of the number; if the answer is divisible by 7 (including 0), then the number is also.
8	If the last three digits form a number divisible by 8, then so is the whole number.
9	If the sum of the digits is divisible by 9, the number is also.
10	If the number ends in 0, it is divisible by 10.

Class Activity 1: Diagramming Divisibility

Place the numbers that are at the bottom of the page in the correct circle. If the number has something in common with two circles, place it in the space that is common to both; if it has something in common with all three circles, place it in the space that indicates this. If the number has nothing in common with any of the factors, place it outside the circles.



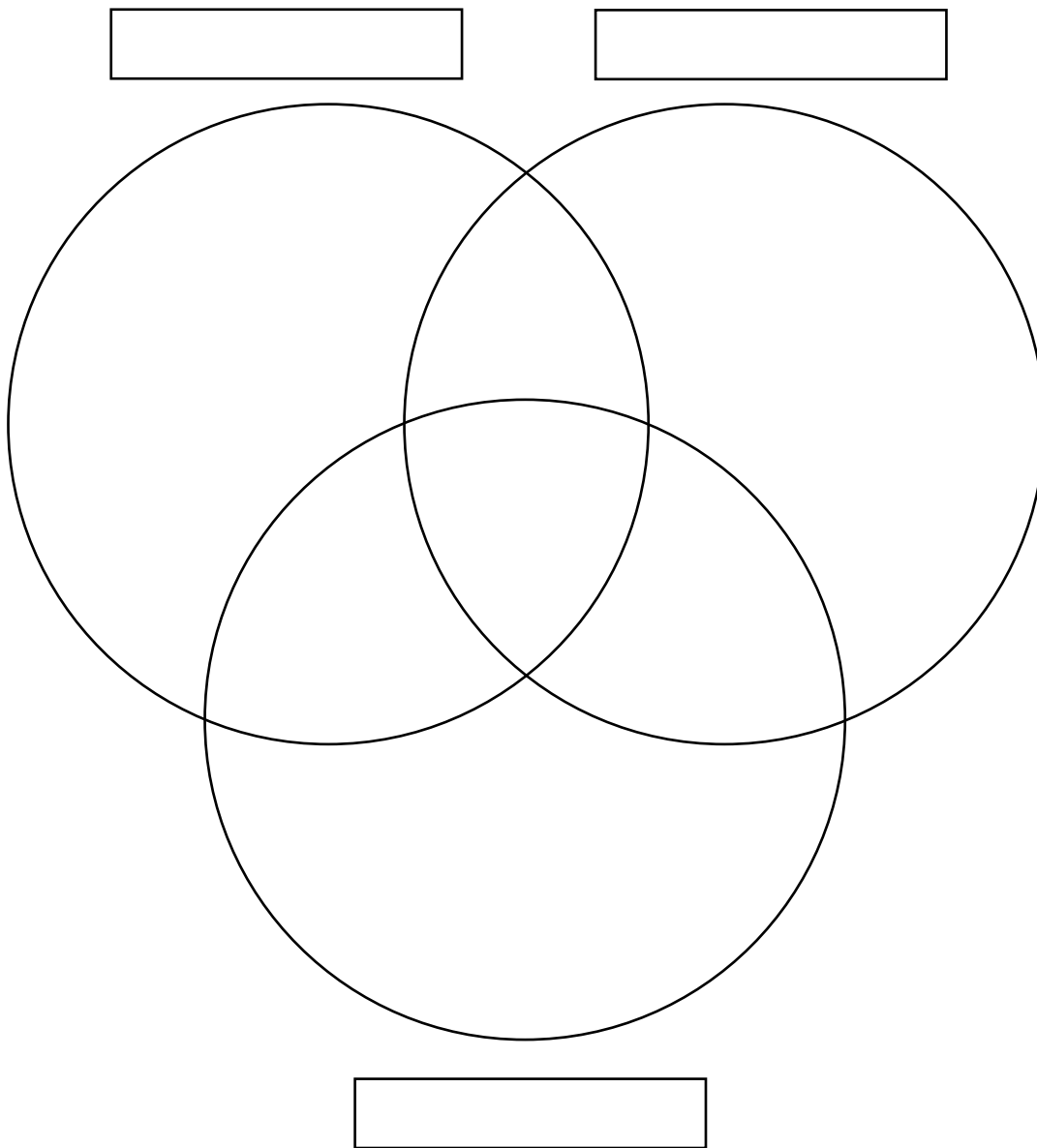
4	6	8	10	12
13	14	15	18	22
23	24	25	28	30
33	35	40	54	59
60	66	90	111	115

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Class Activity 2: Diagramming Divisibility

Work with your partner and use the Venn diagram below to design your own Diagramming Divisibility activity sheet. Be sure to create an answer key so when you share it with the class you can help with the solutions.



Factor Trees

Direct Instruction

Explain to students that many math processes call for finding the factors of numbers—numbers that can be multiplied to produce the original number. Some call for finding the greatest common factor, or GCF, of two numbers. Some call for the prime factors of a number.

There are several ways to find factors. One that can help visually keep track of all the factors is called a factor tree. This is a diagram with a vaguely treelike shape. It uses “branches” to show the factors of a number.

Before looking any further at factor trees, review some terms. Students probably know that all whole numbers other than 1 can be written as the product of factors. A prime number is a number that has only two factors, itself and 1. An example of a prime number is 13. Its only factors are 13 and 1. A composite number is a number that has more than two factors. An example of a composite number is 6. Its factors include 6, 3, 2, and 1. Prime factors are factors that are also prime numbers. The greatest common factor (GCF) of two numbers is the largest number that is a factor of both numbers.

Talk students through the process of making a factor tree. To create a factor tree, start by writing the number to be factored on the board, or on a sheet of poster paper. The “tree” will spread out, so leave plenty of space on both sides. The larger the number you are factoring, the more space you will need.

Think of two factors of the number. Write the factors below and a little to one side of the original number. Try to keep them on the same level as each other, otherwise your factor tree can get confusing as it goes on. Draw a short diagonal line to connect each factor to the original number.

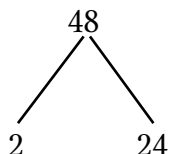
One easy way to start finding factors is to start with the smallest prime numbers and see if any of them are factors. The smallest primes are 2, 3, 5, and 7. Here are some easy tricks to see if 2, 3, or 5 are factors of a number:

- If the last digit of a number is either 0 or an even number, the number is divisible by 2, so 2 is a factor.
- If the sum of the digits of a number is divisible by 3, the number is divisible by 3, so 3 is a factor.
- If the last digit of a number is either 0 or 5, the number is divisible by 5, so 5 is a factor.

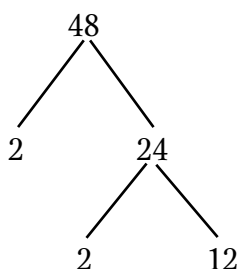
For example, we can look at the number 30 and say immediately that it is divisible by 2 (last digit is 0), 3 (the sum of the digits is 3, which is divisible by 3), and 5 (the last digit is 0), so all three of these numbers are factors of 30.

Instruction

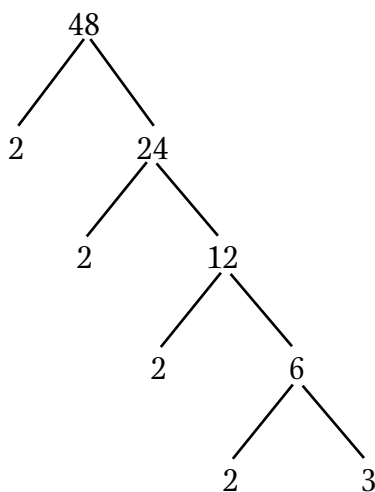
Tell students to try this approach with the number 48. Ask students if 2 is a factor of 48. The last digit of the number is 8, which is an even number. This means that 2 is a factor. To find the other factor that, when multiplied by 2, equals 48, divide 48 by 2. $48 \div 2 = 24$. We can write 2 and 24 as factors of 48. Write the factors below the original number. Connect each factor to the original number with short diagonal lines.



Next, ask students to look to see if these factors can be broken down any further. Tell them we know that 2 is a prime factor because it has only 2 factors, 1 and itself. What about 24? Again, start by seeing if 24 is divisible by 2. The last digit—4—is an even number, so 24 is divisible by 2. $24 \div 2 = 12$. We can write 2 and 12 as factors of 24.



Continue this process, finding factors for each factor, until all your factors are prime numbers.



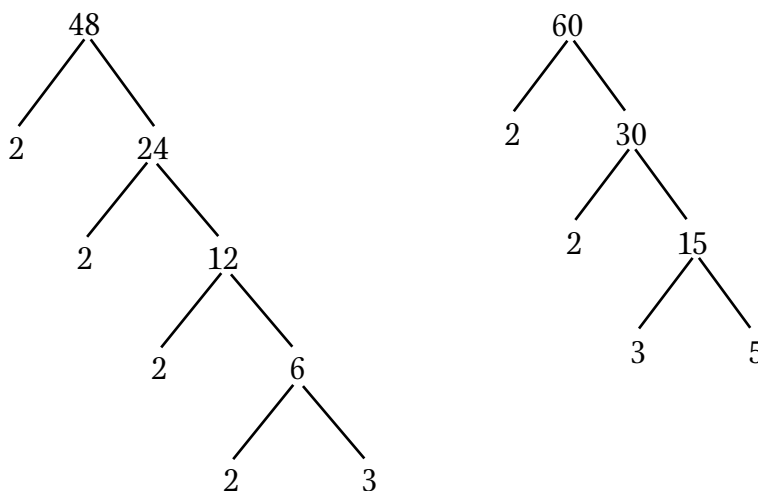
Ask students if any of these factors be broken down further. No, they are all prime numbers. We have factored 48 as far as we can.

Instruction

Tell students the next step is to collect the factors from each step. Any number that is at the end of a “branch” in the “tree” is a prime factor. In this case, the prime factors of 48 are $2 \times 2 \times 2 \times 2 \times 3$.

Explain that when we have repeated multiplication of the same factor, it is customary to use exponents as a kind of shorthand. In the case of the prime factors of 48, 2 is a factor four times. We can write this as 2^4 . We can then say that 48 is the product of $2^4 \times 3$.

We can use prime factors to find the greatest common factor (GCF) of two numbers. Let’s say we need to find the GCF for 48 and 60. Look at the factor trees for both numbers.



The prime factors of 48 are $2 \times 2 \times 2 \times 2 \times 3$. The prime factors of 60 are $2 \times 2 \times 3 \times 5$. What prime factors do they have in common? Both include two 2s and one 3. This means that the GCF of both numbers is $2 \times 2 \times 3$, or 12.

Review the steps in creating a factor tree.

1. Write the number to be factored.
2. See if 2 is a factor of the number. If it is, write 2 a little below and to one side of the number. Connect it to the number with a short diagonal line. Divide the number by 2 to find the other factor that, when multiplied by 2, produces the number. Write it below the number in the same way you wrote 2. If 2 is not a factor, try the other small primes: 3, 5, and 7.
3. Repeat step 2 on the factors you identified in step 2. Write the new factors in the same way, connecting them to the number they factor with short diagonal lines.
4. Continue factoring until all your factors are prime numbers.
5. Collect the factors, and write them using exponential notation to show repeated multiplication of the same factor.

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Use these factor trees to answer the questions. Add or delete lines as needed. Remember, 2 is a factor of a number if the last digit is either 0 or an even number. Three is a factor if the sum of the digits is divisible by 3. Five is a factor if the last digit is either 0 or 5.

